A Mathematical Model of Mongolian Livestock Populations

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Swiss Meeting for Infectious Disease Dynamics
St. Gallen, Switzerland, 30 August 2012

Swiss TPH

- Subsistence herding is an important component of Mongolian livestock production.
- Herding is threatened by
  - Pasture degradation due to overpopulation
  - Extreme weather (dzuds)
  - Infectious diseases.
- Develop mathematical models to include these effects on population dynamics.
- Allow for rational planning of livestock management in Mongolia.

- Use difference equations with a time step of one year.
- Extend the Leslie-Gower competition model.
- Model four species that compete with each other
  - Goats
  - Sheep
  - Cattle
  - Horses.
- Stratify each species by age and gender.
- Model includes
  - Constant per-capita birth
  - Density-dependent survival
  - Density-independent culling
  - Migration of new animals.

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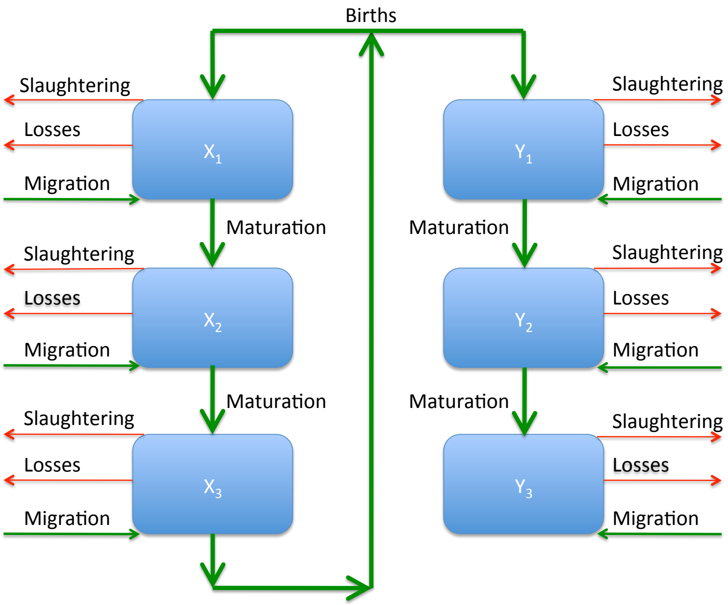
Schematic of Goat Population Dynamics



X_1: New born females
X_2: Juvenile females
X_3: Adult females
Y_1: New born males
Y_2: Juvenile males
Y_3: Adult males

Swiss TPH

$$X_1(t+1) = bX_3(t) + \frac{mX_1(t)}{G(t)}$$

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Equations for Goat Population Dynamics

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- Model was calibrated to data for Töv aimag from published annual Mongolian statistics and detailed herd and survey data.
- Model parameters were fit for different time periods where parameters were assumed to be constant within periods but could change between periods.

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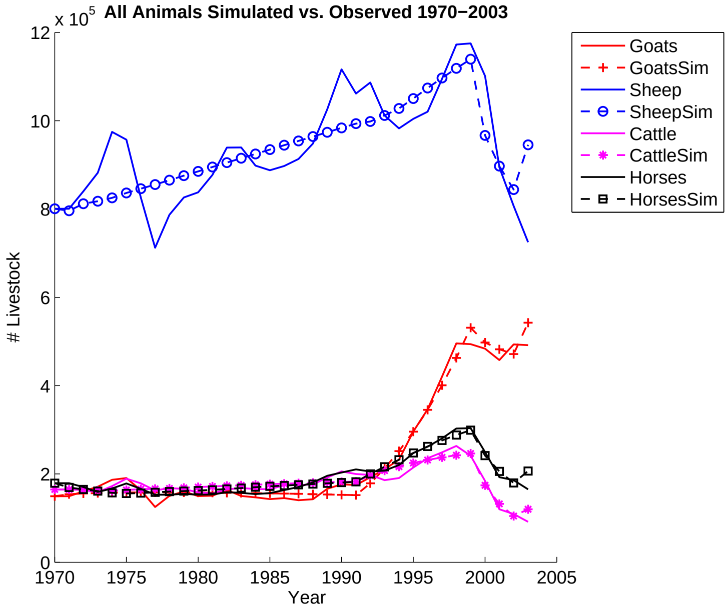
- From 1970–2010 species population and migration data were taken from the National Statistical Office of Mongolia.
- Animal counts from 23 herds in 2011 provided estimates of age and sex distributions.
- Herder surveys in 2011 provided data on culling and animal losses from natural causes.

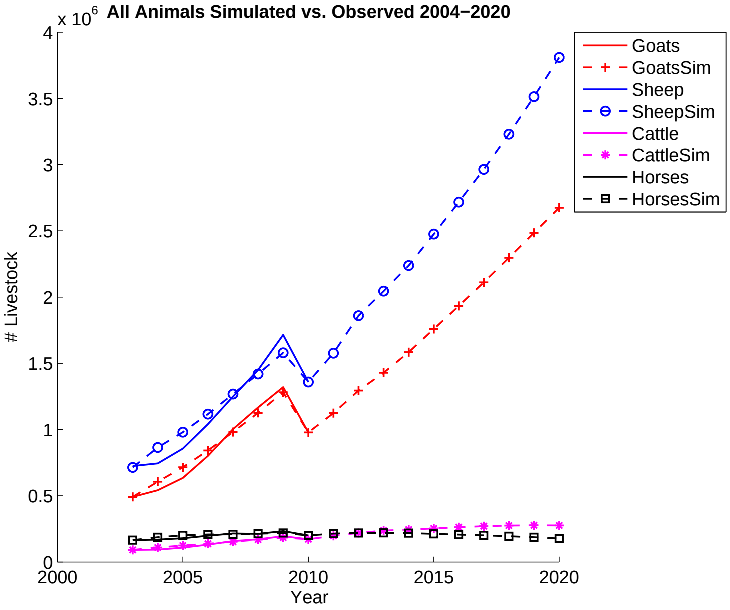
Label	Period	Description
A	1970–1991	Socialist
B	1992–1999	First post-socialist growth
C	2000–2002	First dzud
D	2003	First post-dzud recovery
E	2004–2009	Second growth
F	2010	Second dzud
G	2011	Second post-dzud recovery
H	2012–2020	Future predictions

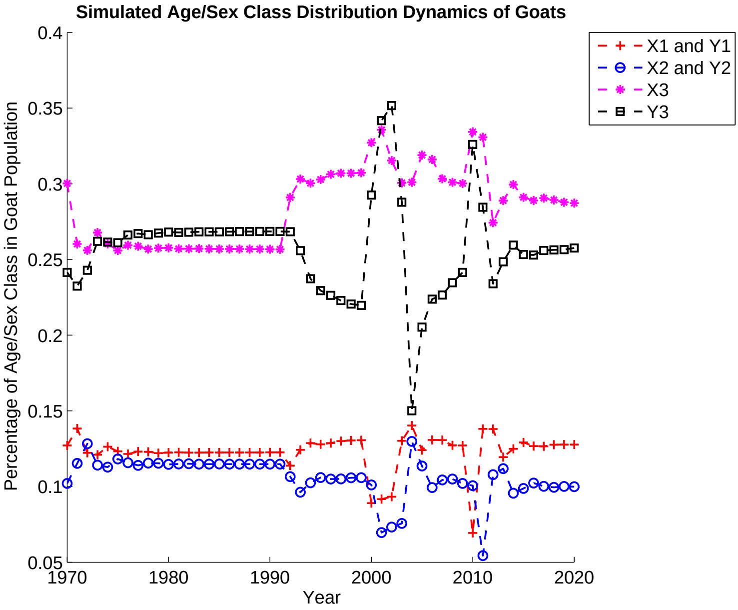
Simulation of Total Population Size from 1970–2003



Simulation of Total Population Size from 2003–2020

Swiss TPH




Simulated Age/Sex Class Distribution Dynamics of Goats
Percentage of Age/Sex Class in Goat Population
Year
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
1970
1980
1990
2000
2010
2020
X1 and Y1
X2 and Y2
X3
Y3

- We can reproduce the data on Mongolian livestock populations.
- The model can be used to help plan for sustainable livestock development that is more resistant to dzuds.
- The model can be extended to include the transmission dynamics of infectious diseases such as brucellosis.